Due: before the end of the semester. If you have any questions or confusion, please just contact me.

1. For the following three models: What are their symmetry classes? Give the relevant symmetry operators. When are these models topologically non-trivial? Calculate either the Zak-Berry phase, the first Chern number, or the $\mathbb{Z}_2$ invariant based on the parity, as you think is appropriate or possible.

   (a) The SSH model:

   $$H = -t\sum_j c_j^\dagger c_{j+1}\left(1+\delta \mathrm{e}^{\mathrm{i}\pi j}\right) + \text{H.c.}\,.$$

   What happens if you change the global phase of your eigenstates? What happens if you change the definition of your unit cell by shifting it one atom to the left or right?

   (b) The 1D Kitaev model:

   $$H = -\sum_j \Psi_j^\dagger \mu \boldsymbol{\sigma}^z \Psi_j + \sum_j \Psi_j^\dagger \left[\Delta \mathrm{i}\boldsymbol{\sigma}^y + -t\boldsymbol{\sigma}^z\right]\Psi_{j+1} + \text{H.c.}\,,$$

   where $\Psi_j^\dagger = \{c_j^\dagger, c_j\}$ with $c_j^{(\dagger)}$ annihilating (creating) a spinless particle at site j.

   (c) The 2D Kitaev model on a square lattice:

   $$H = -\sum_j \Psi_j^\dagger \mu \boldsymbol{\sigma}^z \Psi_j + \sum_{\langle i,j\rangle} \Psi_i^\dagger \left[\Delta\left([\vec{\delta}_{ij}]^x \mathrm{i}\boldsymbol{\sigma}^y + [\vec{\delta}_{ij}]^y \mathrm{i}\boldsymbol{\sigma}^x\right) - t\boldsymbol{\sigma}^z\right]\Psi_j\,,$$

   where $\Psi_j^\dagger = \{c_j^\dagger, c_j\}$ with $c_j^{(\dagger)}$ annihilating (creating) a spinless particle at site j. Here $\vec{\delta}$ is the vector between nearest neighbours.

The first step is to Fourier transform and then solve for the eigenstates in a convenient representation. You can set $t = 1$ and lattice spacing $a = 1$ everywhere for convenience. Note that these are lattice Hamiltonians, but you will treat the momenta as continuous variables once you have performed the Fourier transforms.