1. A classical gas in three dimensions is constrained by a wall to move in the $x \geq 0$ region. A potential

$$V(x) = \frac{\alpha x^2}{2}$$

attracts the atoms to the wall. The atoms are free to move in an area A in the y-z directions. If the gas is at uniform temperature T, show using the grand canonical ensemble that the number of particles varies as

$$N(x) = N\sqrt{\frac{2\alpha\beta}{\pi}}e^{-\frac{\alpha\beta x^2}{2}}$$

By considering a slab of gas between x and $x + dx$, show that locally the gas continues to obey the ideal gas law. Hence determine the pressure that the gas exerts on the wall.

2. Determine the density of states for non-relativistic particles in 1 and 2 dimensions assuming that the energy is large enough that the spectrum may be treated as a continuum. How does the density behave for particles on a plane and for particles on a line?

3. Often particles are trapped in a quadratic potential. In d-dimensions, the potential energy felt by a single particle at a position $\vec{r}$ is

$$V(\vec{r}) = \frac{1}{2}\sum_{i=1}^{d}\omega_i^2 r_i^2$$

with $r_1 = x$, $r_2 = y$, and so on. Compute the density of states, $g(E)$ in 2 and 3 dimensions assuming that the energy is large enough that the spectrum may be treated as a continuum.
Tip: First determine $G(E)$, the number of states with energy less than E.