1. Calculate the partition function for a quantum harmonic oscillator with energy levels $E_n = \hbar\omega(n + \frac{1}{2})$.

   (i) Find the average energy E and entropy S as a function of temperature T.

   (ii) A simple model of a solid, due to Einstein, treats the atoms vibrating as a set of N harmonic oscillators. Calculate the heat capacity for $K_B T \gg \hbar\omega$ and $T \to 0$. Compare this to the experimentally known results, for $K_B T \gg \hbar\omega$ one finds $C_V = 3Nk_B$ and for $T \to 0$ one finds $C_V \sim T^3$.

2. We can find a unified way of thinking about various ensembles. Let's start form the Gibbs formula for entropy:

$$S = -k_B \sum_n p(n) \ln p(n) \, .$$

   We can find the microcanonical and canonical ensembles by maximising this with respect to different constraints for $p(n)$.

   (i) Find the microcanonical ensemble for $p(n)$ by adding a constraint that $\sum_n p(n) = 1$ and only states with energy E have non-zero $p(n)$. (To maximise with this constraint use a Lagrange multiplier.)

   (ii) For the canonical ensemble we must add the constraint that the average energy is fixed $\langle E \rangle = \sum_n p(n) E_n$.

   (iii) What happens if we add the constraint that the average energy is fixed $\langle E \rangle = \sum_n p(n) E_n$ and the average particle number is fixed $\langle N \rangle = \sum_n p(n) N_n$?