Preliminaries:

- $\cup$: union "and/or"

- $\cap$: intersection "and"

- $p(A)$: probability of A

- B^*: not B

1. Sketch and shade Venn diagrams for the following:

    (a) $p(A \cap B)$

    (b) $p(A \cup B)$

    (c) $p(A \cap B^*)$

    (d) $p(A^* \cap B^*)$

    (e) Prove, using a Venn diagram, $p(A \cup B) = p(A) + p(B) - p(A \cap B)$.

2. Consider the probability of two events, A and B. We know that $p(A) = 0.1$, $p(B) = 0.2$, and $p(A \cap B) = 0.05$. Compute the probability that

    (a) either A or B occur,

    (b) neither occur, and

    (c) A occurs but not B.

3. Compute the probability that out of 23 randomly selected individuals, at least two of them have the same birthday. (Assume that all birthdays are equally common, and that there are 365 days in a year.)

4. Let $\rho(x) = \frac{1}{N} e^{-x^2/a^2}$ be a probability distribution for $x \in [0, \infty)$.

    (a) Calculate N.

    (b) Calculate $\bar{x}$ and the root mean square of x.

    (c) What is a?