

1.  $V^{3+}$  has two 3d electrons, calculate  $S$ ,  $L$ , and  $J$  using Hund's rules.
2. We will derive the magnetic susceptibility for permanent moments in a paramagnet, assuming the moments are immobile and independent (the Curie law).

- Magnetisation is

$$M = \frac{N}{Z} \sum_{J_z=-J}^J (-g\mu_B J_z) e^{-\beta E_{J_z}}$$

where  $Z = \sum_{J_z=-J}^J e^{-\beta E_{J_z}}$  is the partition function and  $E_{J_z} = g\mu_B B J_z$  is the energy and  $\beta = 1/k_B T$ .

- Show that

$$M = -\frac{Nk_B T^2}{B} \frac{\partial \ln Z}{\partial T}.$$

- Calculate  $Z$  using the geometric series

$$\sum_{k=0}^n r^k = \frac{1 - r^{n+1}}{1 - r}.$$

- Calculate the magnetisation, and find a simple expression by expanding in the small parameter  $x = g\mu_B B J \beta$ .
- Show that  $\chi = C/T$  with  $C = N g^2 J(J+1) \mu_B^2 \mu_0 \beta / 2$  ( $C$  is the Curie constant.)