1. Show that $\vec{k}$, $\vec{E}$, and $\vec{B}$ are all perpendicular to each other for our solution to the EM wave equation:
$$\vec{E} = \vec{E}_0 \cos\left[\omega t - \vec{k}\cdot\vec{r} + \phi_0\right] .$$
Sketch the behaviour of $\vec{E}$ and $\vec{B}$ as a function of position for some value of $\vec{k}$, $\phi_0 = 0$ and $\omega t = 0, \pi/4, \pi/2$.

2. Starting from $\nabla \times \vec{E} = -\partial_t \vec{B}$ and $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \partial_t \vec{E}$ show that
$$\partial_t \left[\frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}\right] = -\frac{1}{\mu_0}\nabla\cdot(\vec{E}\times\vec{B}) - \vec{E}\cdot\vec{J} .$$
By comparing this with the continuity equation $\partial_t \rho = -\nabla\cdot\vec{J} + \sigma$ interpret the above equation. $\vec{S} = \vec{E}\times\vec{H}$ is called the Poynting vector.

3. Consider two infinitely large perfectly conducting parallel plates in the x-z plane. One at $y = 0$ and one at $y = d$.
   - What are the boundary conditions for the electric and magnetic fields on the plates?
   - Find a solution to the wave equation for this system for $\vec{k} = (0, 0, k)$.
   - Find a solution to the wave equation for this system which is made from two waves with $\vec{k}_1 = k(0, \sin\theta, \cos\theta)$ and $\vec{k}_2 = k(0, -\sin\theta, \cos\theta)$.
   - Which property of the wave equation, and Maxwells equations more generally ensures that the sum of two solutions is still a solution?

4. Show that the energy of a simple dipole $\vec{p}$ in an electric field $\vec{E}$ is $U = -\vec{p}\cdot\vec{E}$.

5. Starting from the potential for a dipole $\vec{p} = p\hat{z}$ find the electric field produced by a dipole. Find a coordinate free representation.