Electrodynamics: Summary of Lectures - Some Useful Formulae

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CONTENTS

NOTATION

$\mathbf{E}$	the electric field
$\mathbf{B}$	the magnetic field
$\mathbf{J}$	current density
ρ	charge density
$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2	permittivity of free space
$\mu_0 = 4\pi \times 10^{-7}$ N/A^2	permeability of free space
$e = 1.60 \times 10^{-19}$ C	charge of the electron
$m_e = 9.11 \times 10^{-31}$ kg	mass of the electron
$c = 3.00 \times 10^8$ m/s	speed of light in a vacuum

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I. MAXWELL'S EQUATIONS AND FUNDAMENTALS

Maxwell's equations:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \oint \mathbf{E} \cdot \mathrm{d}\mathbf{S} = \frac{Q}{\varepsilon_0} \tag{I.1}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \oint \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0 \tag{I.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \oint \mathbf{E} \cdot \mathrm{d}\ell = -\frac{\mathrm{d}}{\mathrm{d}t} \oint \mathbf{B} \cdot \mathrm{d}\mathbf{S} \tag{I.3}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \qquad \oint \mathbf{B} \cdot \mathrm{d}\ell = \mu_0 \oint \mathbf{J} \cdot \mathrm{d}\mathbf{S} + \mu_0 \varepsilon_0 \frac{\mathrm{d}}{\mathrm{d}t} \oint \mathbf{E} \cdot \mathrm{d}\mathbf{S} \tag{I.4}$$

Potentials:

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \tag{I.5}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{I.6}$$

Lorentz force law:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{I.7}$$

Continuity equation:

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0 \tag{I.8}$$

Poisson's equation for the scalar potential:

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \tag{I.9}$$

II. MAXWELL'S EQUATIONS IN MATTER

In matter Maxwell's equations can be written as

$$\nabla \cdot \mathbf{D} = \rho_f \tag{II.1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{II.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{II.3}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f - \frac{\partial \mathbf{D}}{\partial t} \tag{II.4}$$

The auxiliary fields:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \P \tag{II.5}$$

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \tag{II.6}$$

III. MATHEMATICAL PRELIMINARIES

Fundamental theorems:

$$\text{Gradient Theorem:} \quad \int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot \mathrm{d}\mathbf{l} = \mathbf{f}(\mathbf{b}) - \mathbf{f}(\mathbf{a}) \tag{III.1}$$

$$\text{Divergence Theorem:} \quad \int (\nabla \cdot \mathbf{A}) \mathrm{d}\tau = \oint \mathbf{A} \cdot \mathrm{d}\mathbf{S} \tag{III.2}$$

$$\text{Curl Theorem:} \quad \int (\nabla \times \mathbf{A}) \cdot \mathrm{d}\mathbf{S} = \oint \mathbf{A} \cdot \mathrm{d}\mathbf{l} \tag{III.3}$$