1. By considering the two particle wavefunction $\psi(\vec{r}_1, \sigma_1; \vec{r}_2, \sigma_2)$:
   (a) Show that in 3 dimensions $\psi(\vec{r}_1, \sigma_1; \vec{r}_2, \sigma_2) = \pm\psi(\vec{r}_2, \sigma_2; \vec{r}_1, \sigma_1)$. What happens if space is 2D?
   (b) If $\psi(\vec{r}_1, \sigma_1; \vec{r}_2, \sigma_2) = -\psi(\vec{r}_2, \sigma_2; \vec{r}_1, \sigma_1)$ and the wavefunction can be factorised into single particle functions $\psi_{1,2}(\vec{r}, \sigma)$ write down $\psi(\vec{r}_1, \sigma_1; \vec{r}_2, \sigma_2)$ in terms of these functions.

2. Consider the one dimensional simple harmonic oscillator,

$$\hat{H}\psi(x) = \left[\frac{\hat{p}^2}{2m} + \frac{m\omega^2 x^2}{2}\right]\psi(x) = \epsilon\psi(x)\,,$$

   where $\hat{p} = -i\hbar\partial_x$ and which has the eigenenergies $\epsilon_n = \hbar\omega(n + 1/2)$ with $n \in \mathbb{W}$.
   (a) Sketch out how this can be solved using an appropriate ansatz for the wavefunction.
   (b) Rewrite the Hamiltonian operator by completing the square on the x^2 and $\hat{p}^2$ terms. Remember that $[x, \hat{p}] \neq 0$!
   (c) By defining operators $\hat{a} = \sqrt{m\omega/2\hbar}(x + i\hat{p}/m\omega)$ and $\hat{a}^\dagger = \sqrt{m\omega/2\hbar}(x - i\hat{p}/m\omega)$ write $\hat{H}$ in terms of these new operators. By comparing this Hamiltonian with the known eigenvalues what must $\hat{a}^\dagger\hat{a}|n\rangle$ be?
   (d) What are $[\hat{a}, \hat{a}^\dagger]$ and $[\hat{a}, \hat{a}]$?
   (e) What are $\hat{a}|n\rangle$ and $\hat{a}^\dagger|n\rangle$?
   (f) If $\hat{a}|0\rangle = 0$ how can we write $|n >$ using $\hat{a}$, $\hat{a}^\dagger$, and $|0\rangle$? Hint: try and calculate $\hat{a}^\dagger\hat{a}\hat{a}^\dagger|0\rangle$ and $\hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a}^\dagger|0\rangle$.
   (g) How would one make fermionic equivalents of $\hat{a}$ and $\hat{a}^\dagger$?