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Summary of Professional Achievements

Contents

1 Personal data

Name:	Nicholas Sedlmayr
Date and Place of Birth:	10.04.1980, Shoreham-by-Sea, Great Britain
Work Address:	Institute of Physics Maria Curie-Skłodowska University pl. Marii Curie-Skłodowskiej 1 20-031 Lublin, Polska
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2 Education

2006 Ph.D., Theoretical Physics, The University of Birmingham, UK
Working under the supervision of Prof. Igor Lerner
Title of the doctoral thesis: *"The Coulomb Blockade in Quantum Dots and a Metamagnetic Quantum Critical Point"*
Confirmation of obtaining the academic degree by the Ministry of Science and Higher Education: Warsaw, 10.02.2017

2002 MSci. First class with honours
Equivalent to Polish: Mgr, oceny "bardzo dobr" z wyróżnieniem
Theoretical Physics and Applied Mathematics
The University of Birmingham, UK
Received the Moreton Prize for Second Best Student graduating in 2002
Confirmation of obtaining the academic degree by the Ministry of Science and Higher Education: Warsaw, 10.02.2017

3 Employment

2019 - Assistant Professor
Institute of Physics, Maria Curie-Skłodowska University
Working on topological insulators, Majorana bound states, and quantum dynamics.

2017 - 2019 Assistant Professor
Faculty of Mathematics and Applied Physics
Rzeszow University of Technology
Working in the Spintronics group on topological insulators, Majorana bound states, and quantum dynamics.

2015 - 2017 Research Associate and Lecturer
Institute for Mathematical and Theoretical Physics
Michigan State University, East Lansing, U.S.A.
Working on hybrid structures of topological superconductors and insulators, and performing theoretical description of the experiments of the group of Prof. Stuart Tessmer.

2013 - 2015 Scientific worker
Institute of Theoretical Physics, CEA Saclay, France
Working with Prof. Cristina Bena and Prof. Pascal Simon on the existence of Majorana bound states on the edges of topological superconductors.

2010 - 2013 Scientific worker and tutor
Department of Physics, University of Kaiserslautern, Germany
Working with Jun. Prof. Jesko Sirker and Prof. Sebastian Eggert on strongly correlated one dimensional systems, quantum dynamics, and topological insulators.

2007 - 2009 Scientific worker
Institute of Physics, Martin-Luther-University
Halle (Saale), Germany
Working with Prof. Jamal Berakdar on the dynamics and interactions of domain walls.

2006 - 2007 Scientific worker
Max-Planck Institute for Microstructure Physics
Halle (Saale), Germany
Working with Prof. Jamal Berakdar on the dynamics and interactions of domain walls.

4 Scientific achievements according to art. 219 ust. 1 pkt. 2 Ustawy

4.1 Summary information on scientific publications

Since my PhD I have been the co-author of 30 articles published in peer reviewed scientific journals, 3 conference proceedings, 1 set of lecture notes, and 1 (on-line) book chapter.

Scientific publications included in the Web of Science database: (retrieved on 23/01/2020)

- Hirsch Index (h-index): 11
- Total number of publications (articles): 36
- Total number of citations: 283
- Total number of citations without self-citations: 216
- Average number of citations per article: 7.86

Scientific publications included in the Google Scholar database: (retrieved on 23/01/2020)

- Hirsch Index (h-index): 11
- Total number of citations: 348

4.2 List of articles comprising the scientific achievement according to art. 219 ust. 1 pkt. 2 Ustawy

(H1) F. Gebhard, K. zu Münster, J. Ren, **N. Sedlmayr**, J. Sirker, and B. Ziebarth
Particle injection into a chain: decoherence versus relaxation for Hermitian and non-Hermitian dynamics
Annalen der Physik, **524**, 286 (2012)
5 citations - Impact Factor 3.276 - MNiSW Points 100
My participation consisted in planning and performing the analytical calculations in sections 3, 4, and 5, numerical calculations for the data in figures 2, 3, 4, 5, 6, 7, and 13, analysis and interpretation of the results, and cowriting the article.

(H2) **N. Sedlmayr**, J. Ren, F. Gebhard, and J. Sirker
Closed and open system dynamics in a fermionic chain with a microscopically specified bath: Relaxation and thermalization
Phys. Rev. Lett., **110**, 100406 (2013)
11 citations - Impact Factor 9.227 - MNiSW Points 200
My participation consisted in planning and performing the analytical calculations, data analysis, analysis and interpretation of the results, and cowriting the article.

(H3) J. Sirker, N.P. Konstantinidis, F. Andraschko, and **N. Sedlmayr**
Locality and thermalization in closed quantum systems
Phys. Rev. A, **89**, 042104 (2014)
36 citations - Impact Factor 2.907 - MNiSW Points 100
My participation consisted in planning and performing the analytical calculations in sections II and III, numerical calculations for the data in figure 6, data analysis, analysis and interpretation of the results, and cowriting the article.

(H4) J. Sirker, M. Maiti, N.P. Konstantinidis, and **N. Sedlmayr**
Boundary fidelity and entanglement in the symmetry protected topological phase of the SSH model
Journal of Statistical Mechanics: Theory and Experiment, P10032 (2014)
22 citations - Impact Factor 2.371 - MNiSW Points 70
My participation consisted in planning and performing the analytical calculations in sections 2 and 3, analysis and interpretation of the results, and cowriting the article.

(H5) **N. Sedlmayr**, and C. Bena
Visualising Majorana bound states in 1D and 2D using the generalised Majorana polarisation
Phys. Rev. B, **92**, 115115 (2015)
22 citations - Impact Factor 3.736 - MNiSW Points 140
My participation consisted in all analytical and numerical calculations, analysis and interpretation of the results, and cowriting the article.

(H6) **N. Sedlmayr**, J.M. Aguiar-Hualde, and C. Bena
Flat Majorana bands in 2d lattices with inhomogeneous magnetic fields: topology and stability
Phys. Rev. B, **91**, 115415 (2015)
21 citations - Impact Factor 3.736 - MNiSW Points 140
My participation consisted in planning and performing all analytical calculations, numerical calculations for the data in figures 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, and 17, analysis and interpretation of the results, and cowriting the article.

(H7) **N. Sedlmayr**, M. Guigou, P. Simon, and C. Bena
Majoranas with and without a 'character': hybridization, braiding and Majorana number
Journal of Physics: Condensed Matter, **27**, 455601 (2015)
5 citations - Impact Factor 2.711 - MNiSW Points 70
My participation consisted in developing the ideas in section II, all analytical and numerical calculations, analysis and interpretation of the results, and cowriting the article.

(H8) E. König, A. Levchenko, and **N. Sedlmayr**
Universal fidelity near quantum and topological phase transitions in finite 1D systems
Phys. Rev. B, **93**, 235160 (2016)
9 citations - Impact Factor 3.736 - MNiSW Points 140
My participation consisted in co-devising the project, numerical calculations for the data in figure 4, analysis and interpretation of the results, and cowriting the article.

(H9) M. Guigou, **N. Sedlmayr**, J.M. Aguiar-Hualde, and C. Bena
Signature of a topological phase transition in long SN junctions in the spin-polarized density of states
Europhys. Lett., **115**, 47005 (2016)
7 citations - Impact Factor 1.886 - MNiSW Points 70
My participation consisted in the numerical calculations for the data in figures 2, 5, 6, 7(a,b), 8, and 9, analysis and interpretation of the results, and cowriting the article.

(H10) **N. Sedlmayr**, J.M. Aguiar-Hualde, and C. Bena
Majorana bound states in open quasi-1D and 2D systems with transverse Rashba coupling
Phys. Rev. B, **93**, 155425 (2016)
15 citations - Impact Factor 3.736 - MNiSW Points 140
My participation consisted in co-devising the project, all analytical calculations, the numerical calculations for the data in figures 1, 2, 4, 5, 7, 8, 10, 11, 12, 13, analysis and interpretation of the results, and cowriting the article.

(H11) I.M. Dayton, **N. Sedlmayr**, V. Ramirez, T. Chasapis, R. Loloee, M. Kanatzidis, A. Levchenko, and S. Tessmer
Scanning tunneling microscopy of superconducting topological surface states in Bi_2Se_3
Phys. Rev. B (Rapid Comm.) **93**, 220506(R) (2016)
2 citations - Impact Factor 3.736 - MNiSW Points 140
My participation consisted in the analytical and numerical calculations for figure 4, analytical calculations of the results in equations 4 to 7, analysis and interpretation of the results, and cowriting the article.

(H12) **N. Sedlmayr**, V. Kaladzhyan, C. Dutreix, and C. Bena
Bulk boundary correspondence and the existence of Majorana bound states on the edges of 2D topological superconductors
Phys. Rev. B, **96**, 184516 (2017)
7 citations - Impact Factor 3.736 - MNiSW Points 140
My participation consisted in devising the project, numerical calculations for the data in figures 1, 2, 4, 5, 6, and 7, analysis and interpretation of the results, and cowriting the article.

(H13) **N. Sedlmayr**, P. Jäger, M. Maiti, and J. Sirker
Bulk-boundary correspondence for dynamical phase transitions in one-dimensional topological insulators and superconductors
Phys. Rev. B, **97**, 064304 (2018)
12 citations - Impact Factor 3.736 - MNiSW Points 140
My participation consisted in planning and performing the analytical calculations in sections II and III, numerical calculations for the data in figures 1, 7, 9, 10, 12, 13, and 15, development of the theoretical ideas in sections IV and V, data analysis for the data presented in figures 2, 6, and 8, analysis and interpretation of the results, and cowriting the article.

(H14) **N. Sedlmayr**, M. Fleischhauer, and J. Sirker
Fate of dynamical phase transitions at finite temperatures and in open systems
Phys. Rev. B, **97**, 045147 (2018)
13 citations - Impact Factor 3.736 - MNiSW Points 140
My participation consisted in planning and performing the analytical calculations of the results in sections II and III, checking of the analytical results in section IV, development of the key ideas in section II, analysis and interpretation of the results, and cowriting the article.

4.3 A description of the scientific achievement that is the basis of the habilitation: The Topology and Dynamics of Quantum Systems

4.3.1 Background

The field of the topology and dynamics of quantum systems brings together several very important strands of thinking in modern physics, and the work collected here has made contributions to several of these.

Depending on the parameters of a material, or external conditions, for example its temperature or the strength of an applied magnetic field, the same material can be in various phases, and changing the parameters can cause the phase to change at a phase boundary in parameter space. The phase boundary divides the parameter or phase space of the system into regions with different phases. Traditionally phases of matter in physics have been understood in terms of Landau's classification based on different symmetries being present either side of the phase boundary [1]. In this scheme phases are identified by order parameters which show whether a phase possesses a particular symmetry, a classic examples being the ferromagnetic to paramagnetic transition as a function of applied magnetic field. In this case the order parameter is the total magnetisation. Two recent ideas that do not follow this scheme have widened considerably our scope for the classification of phases: topological phases [2] and dynamical phase transitions [3, 4].

The groundstates of band insulators and superconductors can posses topological properties. In this case the topological robustness is assured by the existence of a bulk gap above the groundstate. Arbitrary modifications of the system which change the band structure but do not close the gap, cannot change its topological properties. For a symmetry protected topological phase transition it is the topological properties of the ground state which change across a phase boundary, whereas the symmetry properties remain the same. A topological phase is labelled by an appropriate topological index, and it is this that changes at the phase boundary, signalling a phase transition. Therefore the phase boundary must also be accompanied by a closing of the bulk gap, as only in this way can the topological index change. Strictly speaking the systems must also possess definite symmetry properties, and the exact types of symmetry play a role in the possible topological indices allowed in different dimensions[5, 6]. The symmetries under consideration are three, normally referred to as particle-hole $\mathcal{C}$, time-reversal $\mathcal{T}$, and chirality $\mathcal{V} = \mathcal{C}\mathcal{T}$. For a given Hamiltonian H then particle-hole symmetry requires $\{H, \mathcal{C}\} \equiv H\mathcal{C}+\mathcal{C}H = 0$ with $\mathcal{C}^2 = \pm 1$. Time-reversal symmetry requires $[H, \mathcal{T}] \equiv H\mathcal{T} - \mathcal{T}H = 0$ with $\mathcal{T}^2 = \pm 1$, and chirality requires $\{H, \mathcal{V}\} = 0$.

Phase transitions between different phases in Landau's scheme are accompanied by non-analyticities in the free energy. By contrast dynamical phase transitions occur when there are non-analyticities at certain times in the Loschmidt amplitude in non-equilibrium situations. The Loschmidt amplitude can be thought of as a generalised partition function, which normally contains information about the free energy describing the equilibrium phases.

Presented here are not only both the nature of the topological phases, and quantum dynamics and dynamical phase transitions, but also the way in which these topics combine. In particular the combination of topological and dynamical phase transitions, i.e. dynamical phase transitions for topological insulators and superconductors has been focused on in recent years. Symmetry, topology, and dynamics are all central topics in modern condensed matter physics.

4.3.2 Topologically Protected States and Majoranas

One of the main reasons for the interest in topological insulators and superconductors is the existence of special highly robust states on the surfaces (in the case of three dimensional systems), on the edges (in the case of two dimensional systems), or at the ends (in the case of one dimensional systems). In a system where the topology is protected by a superconducting gap, rather than an insulating gap, these special states are thought to include Majorana bound states [7, 8]. Majoranas are particles which behave as their own anti-particle, and they have some exotic properties, potentially of use in building robust quantum computers [9]. For the condensed matter systems under consideration here, the particle to antiparticle transformation is replaced by the particle-hole transformation. These topologically protected edge states appear in topological phases with non-zero topological indices. Crossing a topological phase boundary into a phase with index zero, a "topologically trivial phase", and these robust edge states will no longer be present.

Majorana bound states appear as topologically protected edge states in superconductors, in particular in superconductors which posses the correct intrinsic or effective superconducting pairing potential. The superconducting pairing potential is the attractive interaction which binds together electrons into so-called Cooper pairs, which are the origin of the superconducting behaviour. Due to the exchange statistics of the electrons bound together, the pair must be odd either under exchange of their orbital or of their spin labels. For a topological superconductor a particular type of pairing is required called p-wave pairing where the orbital part is odd. For s-wave pairing, typical for superconducting metals, it is rather the spin part which is odd. The behaviour and properties of these Majorana bound states are not only of fundamental interest, but also their unusual braiding properties when they are exchanged may have applications for quantum computing [10, 11, 12].

In reference (H7) it has been studied whether Majorana bound states can be characterised into definite different types, and in the case when it can be done, what this can tell us about their properties. In some cases, when the right symmetries are present, the Majoranas can be labelled 'A' or 'B'. This label classifies the Majorana states into two groups within each of which the Majoranas can not destroy each other. I.e. only a Majorana type A can destroy a Majorana type B. This allows for multiple Majorana states to exist on a single edge of a system, provided they are all of the same type. It was proven that this character can only be the chirality of the Majorana states, if there is no chiral symmetry then no character is possible. The consequences of the presence of this character, or the absence of this character, on the braiding and scattering properties of the Majorana states was also explored. While it protects the Majoranas from some scattering processes it does not seem to affect their braiding properties. The generic situation is that if there are multiple Majorana states present then any two coming into contact with each other will destroy themselves, and they will recombine into finite energy fermionic states.

In Refs. (H5) and (H10) a method to directly visualise the Majorana states in real space was developed, see figure 1 for examples. The Majorana bound states in a topological superconductor are all by definition eigenstates of the particle-hole transformation operator. Therefore a Majorana state localised inside a spatial region $\mathcal{R}$ must satisfy

$$C = \frac{\left|\sum_{\vec{r}\in\mathcal{R}} \langle\Psi|\mathcal{C}\hat{r}|\Psi\rangle\right|}{\sum_{\vec{r}\in\mathcal{R}} \langle\Psi|\hat{r}|\Psi\rangle} = 1\,, \tag{1}$$

where $\hat{r}$ is the projection onto site $\vec{r}$. By directly graphing the local expectation value of this operator for different states:

$$c(\vec{r}) = \langle\Psi|\mathcal{C}\hat{r}|\Psi\rangle\,. \tag{2}$$

one can see in what way Majorana states are formed as the phase boundaries are crossed, and in what way they are robust to changes in the underlying physical system. This we refer to as the Majorana polarisation.

Here the focus is on two different lattice models for a topological superconductor, one spinless and one spinfull. The spinfull Hamiltonian is written in the Nambu basis: $\Psi^\dagger_{\vec{r}} = \{c^\dagger_{\vec{r}\uparrow}, c^\dagger_{\vec{r}\downarrow}, c_{\vec{r}\downarrow}, -c_{\vec{r}\uparrow}\}$, where $c^{(\dagger)}_{\vec{r}\sigma}$ annihilates (creates) a particle of spin σ at site $\vec{r} = (i,j)$ in a square lattice. The spinfull Hamiltonian is

$$\begin{aligned} H &= \sum_{\vec{r}} \Big[\Psi^\dagger_{\vec{r}} \left(-\mu\boldsymbol{\tau}^z - \Delta\boldsymbol{\tau}^x + B\boldsymbol{\sigma}^z\right)\Psi_{\vec{r}} \\ &\quad + \Psi^\dagger_{\vec{r}}\left(-t - \mathrm{i}\alpha\boldsymbol{\sigma}^y\right)\boldsymbol{\tau}^z\Psi_{\vec{r}+\hat{x}} + \mathrm{H.c.} + \Psi^\dagger_{\vec{r}}\left(-t + \mathrm{i}\alpha\boldsymbol{\sigma}^x\right)\boldsymbol{\tau}^z\Psi_{\vec{r}+\hat{y}} + \mathrm{H.c.}\Big]\,, \end{aligned} \tag{3}$$

t is the nearest neighbour hopping, μ the chemical potential, B the applied magnetic field, Δ a proximity induced s-wave superconducting pairing, and α is the strength of the Rashba spin-orbit coupling. $\vec{\boldsymbol{\tau}}$ denotes the Pauli matrices in the particle-hole subspace and $\vec{\boldsymbol{\sigma}}$ the Pauli matrices in the spin subspace. The combination of s-wave pairing and Rashba spin-orbit coupling leads to an effective p-wave pairing. A spinless model with p-wave pairing, motivated by Kitaev's model [8] is

$$H = \sum_{j} \Psi_j^\dagger \mu \boldsymbol{\tau}^z \Psi_j + \sum_{\langle i,j \rangle} \Psi_i^\dagger \left[\Delta \left([\vec{\delta}_{ij}]^x \mathrm{i} \boldsymbol{\tau}^y + [\vec{\delta}_{ij}]^y \mathrm{i} \boldsymbol{\tau}^x \right) - t \boldsymbol{\tau}^z \right] \Psi_j \,, \tag{4}$$

where $\Psi_j^\dagger = \{c_j^\dagger, c_j\}$ with $c_j^{(\dagger)}$ annihilating (creating) a spinless particle at site j. Here $\vec{\delta}_{ij}$ is the vector between nearest neighbour sites i and j. Both of these Hamiltonian's can become topologically non-trivial and host Majorana bound states.

By deforming a 2D lattice from a quasi-1D to a 2D structure, for example by changing the number of atomic sites in one direction, one can track the destruction of the edge states associated with one dimension and the formation of those for another by visualising the Majorana polarisation. In general this allows a direct way of theoretically checking to what extent the states that appear in more realistically modelled systems satisfy the criteria to be Majorana bound states, and thus to be of some use in the potential applications, both quantitatively and qualitatively. Due to the relation between these states and the bulk topology of the system it also offers a simple numerical way of determining the topological phase diagram of an arbitrarily complicated model, the only limitation being the computational limitation on diagonalising the system, see figure 2. This can be especially of use for disordered systems where the calculation of bulk invariants is more demanding or not possible. In order to compare the numerically obtained phase diagram with an exact result the previously unknown topological invariant for the quasi-1D Hamiltonian 4 was calculated (H5).

A possible way of creating flat bands of Majoranas on the boundaries of two dimensional systems, with a future view of investigating their mutual interactions, was developed in (H6). Two dimensional systems which can support flat bands of Majorana states at their edges, which are relatively stable with respect to distortions at the boundary on both square and hexagonal lattices were investigated. Thus allowing one to create many Majorana states along a system edge (H6). The systems consist of lattices of Shiba states [13, 14], formed from magnetic impurities on a superconducting substrate, with appropriate magnetic ordering. The Hamiltonian, as previously in the Nambu basis, is

$$H = \sum_{j} \tilde{\Psi}_j^\dagger \left[-\mu \boldsymbol{\tau}^z - \Delta \boldsymbol{\tau}^x \right] \tilde{\Psi}_j - \frac{t}{2} \sum_{\langle i,j \rangle} \tilde{\Psi}_i^\dagger \boldsymbol{\tau}^z \tilde{\Psi}_j + B \sum_{j} \tilde{\Psi}_j^\dagger \hat{n}_j \cdot \vec{\sigma}_{\sigma\sigma'} \tilde{\Psi}_j \,. \tag{5}$$

μ is the chemical potential, t the hopping strength, and Δ the induced superconducting pairing. The two-dimensional lattice is either hexagonal or square. The last term in equation 5 is a local magnetic field of strength B caused by the magnetic impurities. The flat bands are protected by the previously mentioned character of the Majoranas, but not fully by any bulk topology. Topological phase diagrams, and band structures were calculated, which demonstrate the existence of the flat bands holding macroscopic numbers of Majorana bound states. They were found to be relatively stable with respect to distortions at the boundary and changes in the magnetic ordering which gives rise to the topology in these lattices.

Despite a series of promising experiments [15, 16, 17, 18, 19, 20, 21, 22], an unambiguous detection of Majorana states remains an open question and important goal. With this in mind, by investigating the spin texture of Andreev bound states and Majorana states in long superconductor-normal (SN) metal junctions, it was demonstrated that measuring the spin polarisation in the normal region allows one to identify the topological transition (H9) via spin-polarised scanning tunnelling microscopy measurements, see figure 3. In particular, it was found that the spin polarisation exhibits a moderate accumulation close to the SN boundaries. Most strikingly, this accumulation changes sign when crossing the topological transition. There

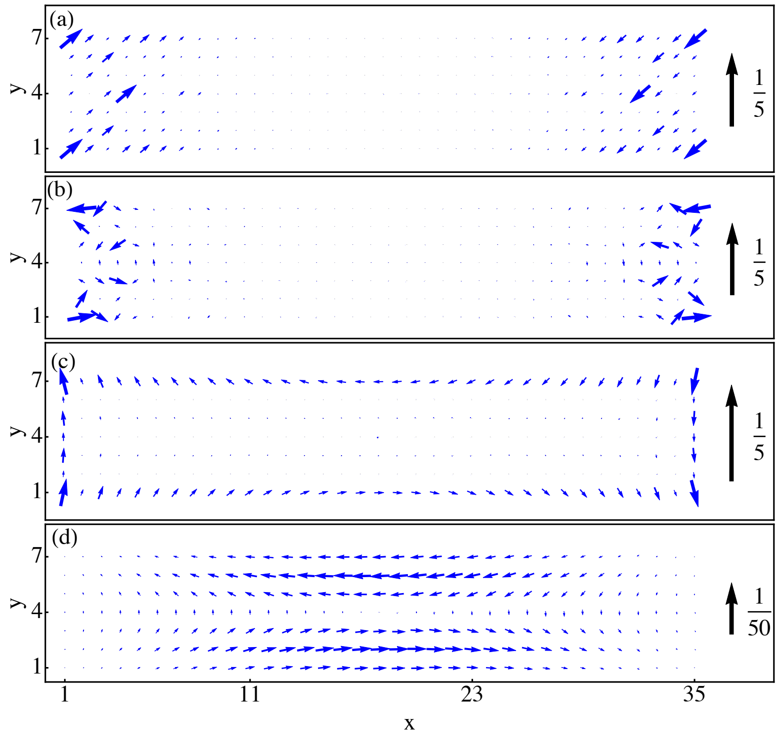

Figure 1: A spatial visualisation of the particle-hole nature of different states in a quasi-1D topological superconductor using the Majorana polarisation, see equations 2 and 3. The size of the arrows is related to the magnitude of the particle-hole overlap of the states, $c(\vec{r})$, and the direction of the arrows to the phase of $c(\vec{r})$, see equation 2. A Majorana bound state must have a uniform phase, and hence all arrows aligned, as can be seen in panel (a) on the left and right. Panel (b) shows non-Majorana edge states, where the state is localised at the end of the wire, but the phase is not aligned. Panel (c) is an intermediate state in which locally the phase is consistent, but not globally for a contiguous state. (d) is a typical bulk state. Based on a numerical simulation of a small 7×35 open lattice. (H5)

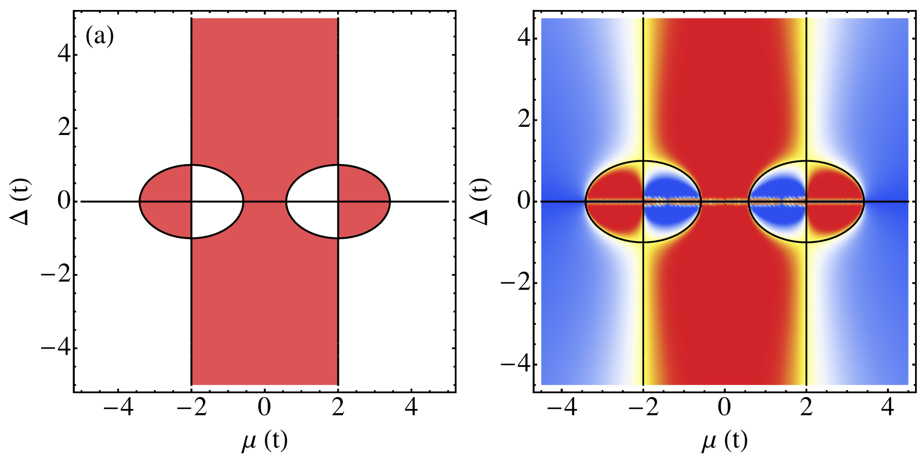

Figure 2: A comparison of the analytical topological phase diagram for a simple spinless quasi-one dimensional topological superconductor (a), and the same obtained from a numerical evaluation of the Majorana polarisation (b) for system of lattice dimensions 3x51. Yellow regions in (b) correspond to the existence of states as seen in figure 1(c). Δ is the strength of the p-wave superconducting pairing and μ is the chemical potential, measured in units of hopping strength t. (H5)

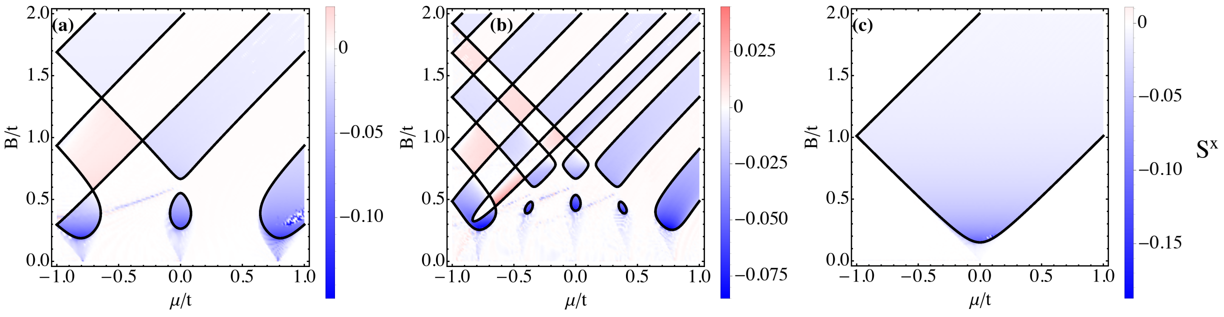

Figure 3: The total spin-polarised density of states orientated parallel to the wires for the lowest-energy state as a function of magnetic field, B, and chemical potential, μ. Dark solid lines show analytically calculated phase boundaries. The full phase diagram obtained is very accurately recovered. The three panels show different wire widths N: (a) $N = 3$, (b) $N = 7$, and (c) the strictly one dimensional limit with $N = 1$. All calculations were performed for a lattice of width of N and a length of 301 lattice sites. See (H9). The Hamiltonian is as for equation 3 with a position dependent pairing term describing the SN junction.

is also a strong asymmetry between the densities of particle and hole states which vanishes as one crosses the topological phase transition.

The concept that relates the bulk topological properties of materials with the existence of protected edge states is the ‘bulk-boundary correspondence’. For two dimensional topological superconductors this is commonly thought to directly relate the topological index characterising the bulk topology and the number of Majorana bound states present on the edges of the system [23]. In fact this is not necessarily correct. Rather the topological index is related to protected bands of edge states which cross the superconducting gap, but these bands do not necessarily have to contain a Majorana bound state (H12). For a Majorana state to be present one requires some additional high symmetry points in the Brillouin zone of the system. This is important if one wishes to really utilise the Majorana states on the edges of a two dimensional system, rather than the bands of edge states. Examples were constructed in which the topological index, in this case a Chern number, was as high as five, but the maximum number of Majorana states which could be formed was only three. The presence or absence of the Majorana states in different phases was demonstrated by a combination of finite size scaling of the energies of the states, and their Majorana polarisation. I.e. a direct consideration of whether they satisfy the definition of a Majorana state. Contrary to the usual interpretation of the bulk-boundary correspondence, the number of Majorana states also depends on the nature of the edges. In reference (H12) zig-zag and armchair edges for a hexagonal lattice were compared, although the number of protected bands is the same in the two cases the number of Majorana bound states contained in them is different. The bulk-boundary correspondence was however found to always apply to the relation between the topological index and the number of topologically protected bands on the edges.

4.3.3 Topological Insulator-Superconductor Heterostructures

One possible method for creating a topological superconductor is by considering topological insulator-superconductor heterostructures [24]. The superconducting proximity effect induced in metals in close contact with a superconductor, whereby the Cooper pairs tunnel into the metal, is well known. However it is less well understood that in a similar way the topologically protected surface states on the surface of topological insulators can leak into appropriate adjoining materials, as has been shown by recent experiments (H11). In these experiments superconducting structures were deposited on the surface of the bulk topological insulator Bi_2Se_3. In (H11) the manner in which the superconductivity and topologically protected surface states interact with each other in these hybrid structures was modelled. Theoretical models were developed which capture the hybridization between the topological surface states and the superconducting states. A very good agreement between the experimental results and the theoretical description supports the model [25].

The density of states of both the topological insulator and the superconductor turn out to exhibit interesting proximity effects and open up new possibilities for observing Majorana states. The superconducting proximity effect in the surface layer of the topological insulator is very long-ranged (H11), meaning that the Cooper pairs which tunnel into the topological insulator surface have a large coherence length. That this phenomenon is caused by the topologically protected surface states was confirmed by a comparison of the disordered metallic case, using Usadel's equation, and by solving Gorkov's equations for the topologically protected surface states in proximity to a superconductor. This rules out the role that could be played by surface states which do not have a topological origin.

The Usadel equation describes a dirty superconductor after disorder averaging in the semi-classical limit. The superconductor is now parameterised by a function $\theta(\vec{r},t)$ which can be related to the superconducting pairing, the local density of states, and other quantities of interest. The standard Usadel equation for a circular superconducting island of radius R surrounded by an infinite normal system is, in position-frequency space,

$$\frac{\partial^2\theta}{\partial x^2}+\frac{1}{x}\frac{\partial\theta}{\partial x}+\frac{i\omega}{E_{\mathrm{Th}}}\sin\theta=0\,. \tag{6}$$

$x=r/R$ is a dimensionless position coordinate and $E_{\mathrm{Th}}=v_F l/2R^2$ is the Thouless energy defined by the Fermi momentum v_F and mean free path l of the dirty metal. The local density of states is

$$\nu(x,\omega)=\nu_0\Re\cos[\theta(x,\omega)], \tag{7}$$

where ν_0 is the density of states of the dirty metal. In a linearised regime, applicable at distances away from the boundary where the proximity effect is weak, an analytical result for equation 6 is

$$\theta(x,\omega)=\theta_0(\omega)\frac{K_0(x\sqrt{i\omega/E_{\mathrm{Th}}})}{K_0(\sqrt{i\omega/E_{\mathrm{Th}}})}\,, \tag{8}$$

where $K_0(z)$ is the modified Bessel function, $\theta_0=\cos^{-1}(\nu_{\mathrm{BCS}}/\nu_0)$ and ν_{BCS}/ν_0 is the BCS density of states in the superconductor. Comparisons of this theory with the experimental data demonstrated that it was not a good model for the long-range proximity effect observed. It was therefore concluded that it is the topologically protected surface states which give rise to the long-range coherence length.

The model for the topologically protected surface states consisted in a two dimensional plane which itself is comprised of the surface states of a three dimensional topological insulator with s-wave pairing in one half of the plain, $x<0$. This was described by Gorkov's equation:

$$\begin{pmatrix}\mathrm{i}\omega_n-H & \mathrm{i}\boldsymbol{\sigma}^y\Delta_x\\ -\mathrm{i}\boldsymbol{\sigma}^y\Delta_x^\dagger & -\mathrm{i}\omega_n-H^*\end{pmatrix}\begin{pmatrix}G_{n,k_y}(x,x')\\ F^\dagger_{n,k_y}(x,x')\end{pmatrix}=\begin{pmatrix}\delta(x-x')\\ 0\end{pmatrix}\,, \tag{9}$$

with the Hamiltonian

$$H^{(*)}\;=\;\frac{v_{Fx}}{2}\begin{pmatrix}0 & \pm\hat{k}_x-\mathrm{i}k_y\\ \pm\hat{k}_x+\mathrm{i}k_y & 0\end{pmatrix}+\mathrm{H.c}\,, \tag{10}$$

describing the linearised 2D surface states [26]. $\Delta_x=\Delta\Theta(-x)$ is the s-wave pairing in the superconducting region with Θ the Heaviside theta function as before and the Fermi velocities in the two regions $v_{Fx}=v_{FS}\Theta(-x)+v_{FT}\Theta(x)$. $G_{n,k_y}(x,x')$ and $F^\dagger_{n,k_y}(x,x')$ are the normal and anomalous Green's functions respectively in the Matsubara representation with fermionic Matsubara frequencies ω_n. From this the local density of states can be calculated:

$$\nu(x,\omega)=\frac{|\omega|\Theta(|\omega|-\Delta_x)}{2\pi v_{Fx}^2}\left[1-J_0\left(\frac{2x\sqrt{\omega^2-\Delta_x^2}}{v_{Fx}}\right)\right]\,, \tag{11}$$

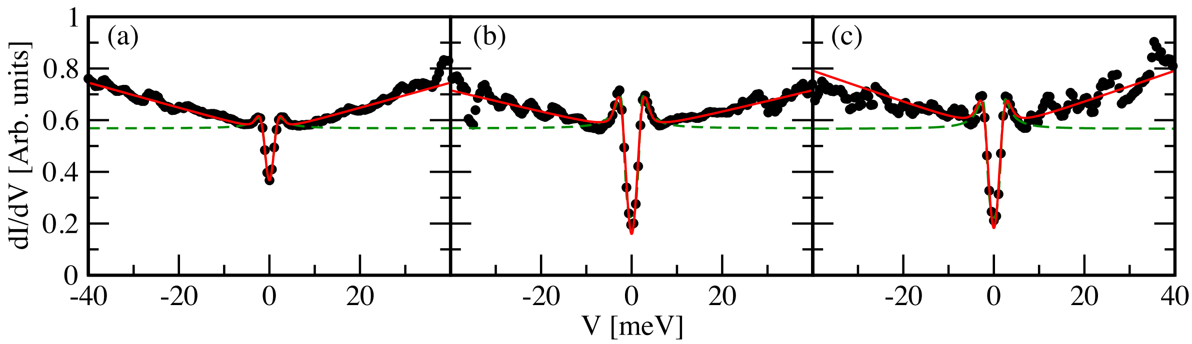

Figure 4: The differential conductance measured by scanning tunnelling microscopy, black dots, for three examples of the superconductor Nb on top of the topological insulator Bi_2Se_3. Fits to the density of states for a hybrid model of superconducting and topological surface states is shown in solid red, and fits to the standard BCS theory are shown in dashed green. As can be seen only the full model correctly captures the structure seen in the experiment [25].

where J_0 is the Bessel function of the first kind. This could then be qualitatively compared with the experimental results from the scanning tunnelling microscopy measurements.

A novel oscillation in the density of states on the surface of the topological insulator in proximity to the superconductor was also observed. This is an oscillation related to, but distinct from, Friedel oscillations which are caused by scattering from density profiles, and Tomasch oscillations [27], which are caused by confinement inside a superconducting pairing potential. These oscillations are qualitatively described by equation 11 with the experimentally appropriate energy scale $v_FT/2x$. The physical origin of the oscillation is from quasiparticle scattering induced by the nonuniform superconducting order parameter. The tunnelling of the topological surface states into the superconductor leads to a distinctive density of states, see figure 4. This demonstrates a mixture of the Dirac cone behaviour of the surface states and the superconducting gap. The hybridization between the topological states and the superconducting states can lead to a new model for topological superconductivity (H11) where Majorana states could potentially be found.

4.3.4 Fidelity and Entanglement of Topological States

Information about topologically protected states can also be obtained by studying the fidelity (the ground-state overlap), the entanglement entropy, and the entanglement spectrum. The fidelity is defined as $F(\delta,\epsilon) = |\langle\Psi(\delta)|\Psi(\delta+\epsilon)\rangle|$ where ϵ parameterises the difference in dimerisation between the two states. The overlap between the two many-body wave functions will vanish exponentially with system size N for $\epsilon \neq 0$ (as in the Anderson orthogonality catastrophe [28]) and therefore it is more appropriate to consider the fidelity density defined as

$$f(\delta,\epsilon) = -\frac{1}{N}\ln F(\delta,\epsilon)\,. \tag{12}$$

The fidelity density has the small ϵ expansion $f(\delta,\epsilon) = \chi(\delta)\epsilon^2 + \mathcal{O}(\epsilon^3)$ where

$$\chi(\delta) = \frac{1}{2}\frac{\partial^2 f}{\partial \epsilon^2}\bigg|_{\epsilon=0} \tag{13}$$

is the fidelity susceptibility which can be used to characterise the phase transition [29, 30]. The entanglement entropy is the von-Neumann entropy of the reduced density matrix of a subsystem of the full system under consideration. In reference (H4) a detailed study of these quantities was performed for a dimerised chain of spinless fermions with open boundary conditions, which is a well-known example for a model supporting a topological phase. This model is known as the Su-Schrieffer-Heeger (SSH) model and was first introduced as a tight-binding model to describe conducting polymers such as polyacetylene [31]. For spinless fermions and a static lattice dimerisation the Hamiltonian is

$$H = -t \sum_{j=1}^{N-1} [1 + (-1)^j \delta](c_{j+1}^\dagger c_j + \text{h.c.}) + U \sum_{j=1}^{N-1} (n_j - 1/2)(n_{j+1} - 1/2) \tag{14}$$

where t is the hopping amplitude, δ the dimerisation, U a nearest-neighbour repulsion, and N the number of lattice sites. The creation operator for a spinless fermion at site j is $c_j^\dagger$, and $n_j = c_j^\dagger c_j$. Firstly the case $U = 0$ was considered. From a scaling analysis of the fidelity susceptibility, $\chi = \chi_0 + \chi_B/N + \mathcal{O}(N^{-2})$, it was demonstrated in (H4) that there is a boundary contribution, χ_B, which is different in the topologically ordered than in the topologically trivial phase. This therefore provides an alternative way of distinguishing the topologically trivial and non-trivial phases which can also be applied to interacting systems in which the single particle analysis is no longer valid.

For the entanglement entropy the system is first divided up into two blocks A and B. Naturally enough the entanglement entropy is a measure of the entanglement between these different parts of the system. It is defined as the von-Neumann entropy of a reduced density matrix

$$S_A = -\text{Tr}(\rho_A \ln \rho_A) \tag{15}$$

with ρ_A the full density matrix for the ground state of the many-body Hamiltonian after a partial trace over B. For the entanglement spectrum and entropy, predictions from massive field theory for a block in the middle of an infinite chain were confirmed. Additionally blocks containing the edge of the chain were considered. For the latter case it was shown that in the topological phase additional entanglement is present, as compared to the trivial phase, which is localised at the boundary. Thus it was demonstrated that these measures offer alternative ways of analysing the presence of topologically protected states in topological insulators and superconductors.

This analysis was then extended to the dimerised chain with a nearest-neighbour interaction and shown that a phase transition into a topologically trivial charge-density wave phase occurs, thus demonstrating that the topological phase persists to finite interaction strengths, see figure 5. This also offers a way of characterising interacting or correlated topological systems where it is either not known how to calculate the appropriate topological index, or where the calculation of the index is not feasible.

Following this detailed analysis of a particular example of a topological insulator in (H8) general quantum phase transitions of the Ising universality class in one-dimensional systems at finite-size systems were considered. This includes such prominent examples as magnetic systems (e.g., spin-Peierls, the anisotropic XY model) and also one dimensional topological insulators of any topologically nontrivial universality class. One can define a rescaled fidelity susceptibility as a function of the only dimensionless parameter LM of such systems, where $2M$ is the gap in the fermionic spectrum and L is the system size. Analytic expressions for the fidelity susceptibility for periodic and open boundaries conditions with zero, one, or two edge states were found (H8). The latter were shown to have a crucial impact and alter the susceptibility both quantitatively and qualitatively. The analytical solutions were supported by a comparison to numerical data on a typical one dimensional topological insulator.

4.3.5 Thermalisation and Dynamics

When, how, and why closed and open quantum systems thermalize still remains an open question. Due to the exponentially many conserved quantities any closed quantum system has, it is not as clear as in the classical case under what conditions a quantum systems thermalises. One idea is the eigenstate thermalisation hypothesis which states that any eigenstate in the thermodynamic limit gives the same result for local observables as a thermal ensemble average [32, 33]. However the reason for this remains unclear. To elucidate thermalisation further a necessary and sufficient condition for the thermalisation of an observable in a closed quantum

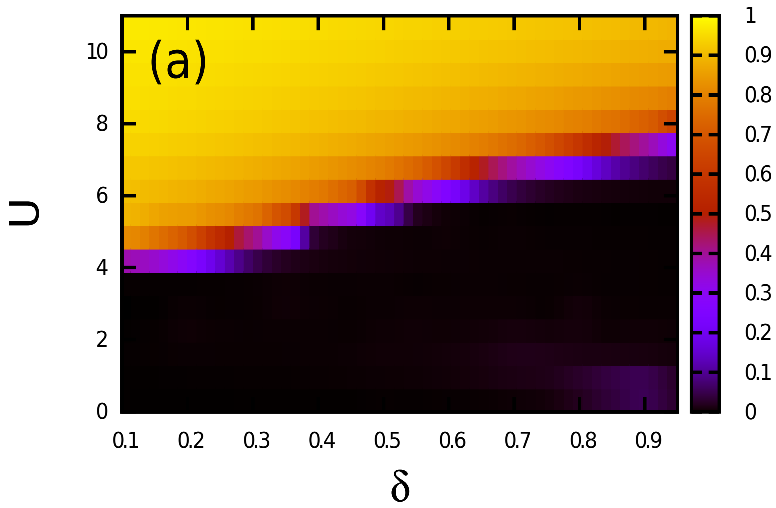

Figure 5: The topological phase diagram for an interacting dimerised one dimensional chain of spinless fermions, where U is the strength of the nearest neighbour interaction, and δ is the amount of dimerisation. The shift from black to yellow signals a change in the degeneracy of the entanglement spectrum groundstate and is an indication of a shift from a topologically nontrivial to a topological trivial phase (H4).

system which does not rely on the eigenstate thermalisation hypothesis was derived (H3). The condition follows from an exact representation of any observable as a sum of a projection onto the local conserved charges of the system, and a projection onto the non-local conserved charges, which is always possible. Local operators are those which do not feature long range coupling terms, where long-range means on the order of the system size, and if they commute with the Hamiltonian then they correspond to a conserved charge. It was shown that thermalisation requires that the time average of the latter part vanishes in the thermodynamic limit while time and statistical averages for the first part are by definition identical. Thus thermalisation was related to a condition on non-local conservation laws, which offers an alternative way of understanding thermalisation which does not rely on the energy eigenbasis which plays no preferential role in the process. Based on numerical data for a one-dimensional spin model as an example the finite-size scaling properties of the thermalisation condition was investigated, finding good agreement with the above condition. Thus the important quantities in the quantum case are not simply the conserved quantities, as for a classical system, but rather the local conserved quantities, which can also have important consequences for transport in quantum systems.

In order to understand the connection between open and closed system dynamics a model system for the injection of fermionic particles from filled source sites into an empty chain was investigated (H1). We studied the ensuing dynamics for Hermitian as well as for a non-Hermitian time evolution in which the particles cannot return to the bath sites (a quantum ratchet). A non-homogeneous hybridization between bath and chain sites permits transient currents in the chain. Non-interacting particles show decoherence in the thermodynamic limit: the average particle number and the average current density in the chain become stationary for long times, whereas the single-particle density matrix displays large fluctuations around its mean value, i.e. as expected without interactions there can be no thermalisation. It was demonstrated, on the other hand, that sizeable density-density interactions between the particles introduce relaxation which is by orders of magnitudes faster than the decoherence processes. Furthermore it was considered under what conditions such a system will thermalise (H2). This allowed for a study of the dynamics of both open and closed systems simultaneously. The time evolution of the observables in the chain after a quantum quench was considered. For low densities it was shown that the intermediate time dynamics can be quantitatively described by a system of coupled equations of motion. For higher densities the results show a pre-thermalisation for the local observables at intermediate times and a full thermalisation to the grand canonical ensemble at long times. For the case of a weak bath-chain coupling it was found, in particular, that the subsystem thermalises despite the bath being apparently oversimplified. Thus thermalisation for both the closed quantum system and for the open subsystem with a rudimentary bath weakly connected could be seen.

4.3.6 Dynamical Phase Transitions

Bringing together all of the above considerations of dynamics and topological phases, the Loschmidt echo for quenches in open one-dimensional lattice models with symmetry protected topological phases was considered. The Loschmidt amplitude measures the overlap between an initial states and its time evolved counterpart:

$$|L(t)| = \left|\langle\Psi_0|\mathrm{e}^{-iH_1t}|\Psi_0\rangle\right| . \tag{16}$$

It can be considered as analogous to the partition function for thermal equilibrium and one can also define a free energy form it which is referred to as the return rate:

$$l(t) = -\frac{1}{N}\ln|L(t)| . \tag{17}$$

When a quantum system is quenched from its ground state, i.e. when a parameter is suddenly changed in the Hamiltonian and the resulting time evolution is considered, then the time evolution can lead to non-analytic behaviour in the return rate at critical times [3]. Such dynamical phase transitions (DPTs) can occur, in particular, for quenches between phases with different topological properties [34].

For quenches in one dimensional topological insulators and superconductors where dynamical quantum phase transitions do occur it was found that cusps in the bulk return rate at critical times are associated also with sudden changes in the boundary contribution (H13). For one main example, once again a dimerised chain of spinless fermions, it was shown that these sudden changes are related to the periodical appearance of two eigenvalues close to zero in the dynamical Loschmidt matrix, the determinant of which gives the Loschmidt amplitude, see figure 6. It was demonstrated, furthermore, that the structure of the Loschmidt spectrum is linked to the periodic creation of long-range entanglement between the edges of the system, see figure 7.

The previous analysis considered only quenches from ground states. Experimentally realisable examples will however be in a low temperature thermal state and may involve the loss of particles during time evolution. Therefore the Loschmidt amplitude generalised to density matrices was determined and results for quenches in closed Gaussian models at finite temperatures as well as for open-system dynamics described by a Gorini-Kossakowski-Sudarshan-Lindblad master equation were obtained. While cusps in the return rate are always smoothed out by finite temperatures it was show that dissipative dynamics can be fine-tuned such that the dynamical phase transitions persist (H14). The smoothing out of the dynamical phase transition by finite temperature places a limit on the possibility of seeing this behaviour in experimental scenarios.

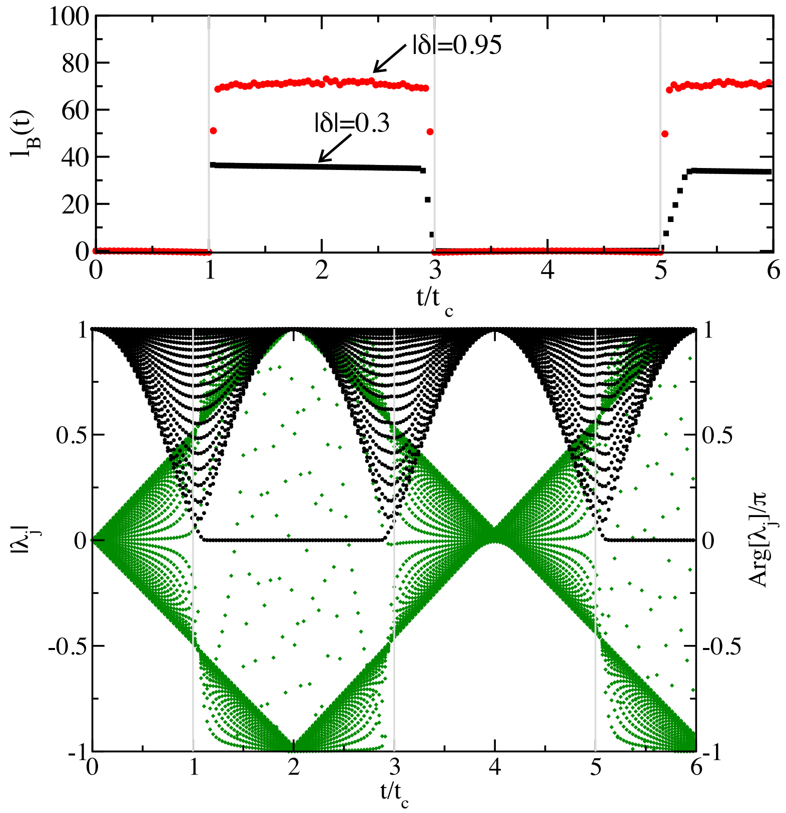

Figure 6: The top panel shows the contribution from the boundaries to the return rate, $l_B(t)$, extracted from a finite size scaling analysis of the return rate for open systems of up to $N = 2200$ sites. The quench is from the trivial into the topological phase, and results for two values of dimerisation are shown. The lower panel shows the absolute value (black circles) and argument (green diamonds) of the eigenvalues of the Loschmidt matrix M for a symmetric quench at $|\delta| = 0.95$ from the trivial into the topological phase for a system of size $N = 80$. From reference (H13). The large boundary contributions are caused by the zero eigenvalues. t_c is there critical time scale at which non-analyticities appear.

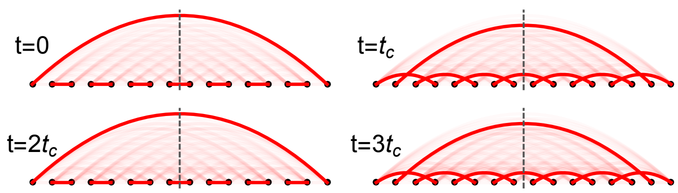

Figure 7: A direct visualisation of the correlations between sites for a quench in a dimerised spinless chain from the topologically nontrivial to the topologically trivial phase (H13). The periodic shifts in long range entanglement between the boundaries can be clearly seen. t_c is there critical time scale at which non-analyticities appear.

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5 Scientific achievements realised across multiple universities, scientific institutes, and abroad

5.1 Invited talks in conferences

2019 Workshop on Recent Developments in the Theory of Topological Systems, Lublin, Poland (12.12 - 13.12)
2019 XIX National Conference on Superconductivity, Bronisławów, Poland (06.10 - 11.10)
2019 Majorana Modes and Beyond, Warsaw, Poland (26.02-27.02)
2018 13th International School on Theoretical Physics: Symmetry and Structural Properties of Condensed Matter (SSPCM 2018), Rzeszów, Poland (10-15.09)
2018 Symposium on the Physics of Majorana Bound States, Warsaw, Poland (05.01)
2013 SFB/TRR 49 Condensed Matter Systems with Variable Many-Body Interactions, Bensheim, Germany (19-20.09)
2011 SFB/TRR 49 Condensed Matter Systems with Variable Many-Body Interactions, Alzey, Germany (15-16.09)

5.2 Invited talks, seminars, and colloquia in universities and scientific institutes

2019 Wrocław University of Technology, Poland (16.10)
2019 Institute of Nuclear Physics, Polish Academy of Sciences, Kraków, Poland (04.06)
2019 Adam Mickiewicz University, Poznań, Poland (15.05)
2019 Johannes Gutenberg University Mainz, Germany (30.04)
2018 Maria Curie-Skłodowska University, Lublin, Poland (20.11)
2017 CEA Saclay, France (25.09)
2016 University of Wisconsin-Madison, Madison, USA (15.11)
2016 University of Manitoba, Winnipeg, Canada (25.11)
2014 Technical University of Kaiserslautern, Germany (06.11)
2012 Marburg University, Germany (08.11)
2011 Max Planck Institute for Solid State Research, Stuttgart, Germany (06.07)
2011 Martin-Luther University, Halle, Germany (02.05)

5.3 Other contributed talks in conferences

2019 APS March Meeting, Boston, USA (04.03 - 08.03)
2018 New Trends in Topological Insulators, Luxembourg City, Luxembourg (16.07)
2017 APS March Meeting, New Orleans, USA (13-17.03)
2016 APS March Meeting, Baltimore, USA (14-18.03)
2013 APS March Meeting, Baltimore, USA (18-22.03)
2012 CMD24-CMMP12, Edinburgh, UK (03-07.09)
2012 DPG Spring Meeting, Berlin, Germany (26-30.03)
2012 APS March Meeting, Boston, USA (27.02-03.03)
2011 CMMP11, Manchester, UK (13-15.12)
2011 APS March Meeting, Dallas, USA (21-25.03)
2010 Joint European Magnetic Symposia, Kraków, Poland (23-28.08)
2009 DPG Spring Meeting, Dresden, Germany (22-27.03)

5.4 Contributed posters at conferences

2019 Bound States in Superconductors and Interfaces, Dresden, Germany (08.04 - 10.04)
2018 10th International School and Conference on Physics and Application of Spin, Linz, Austria (05-09.08)
2014 GDR Physique Quantique Mesoscopique, Aussois, France (01-04.12)
2014 Workshop Topological and Dirac Matter, Bordeaux, France (12-14.11)
2014 Focus Workshop: Topological Matter Out of Equilibrium, Dresden, Germany (27-29.03)
2013 Quantum Many-Body Dynamics in Open Systems, Bad Honnef, Germany (02-05.04)
2012 SFB/TRR 49 Condensed Matter Systems with Variable Many-Body Interactions Annual Retreat, Wiesbaden, Germany (20-21.09)
2012 Workshop on Innovations in Strongly Correlated Electron Systems, Trieste, Italy, (13-17.08)
2011 International Conference on Recent Progress in Many-Body Theories, Bariloche, Argentina (28.11-02.12)
2011 SFB/TRR 49 Condensed Matter Systems with Variable Many-Body Interactions, Alzey, Germany (15-16.09)
2011 Korrelationstage, Dresden, Germany (28.02-04.03)
2010 SFB/TRR 49 Condensed Matter Systems with Variable Many-Body Interactions, Kaiserslautern, Germany (07-08.10)
2010 Quantum Matter in Low Dimensions, Stockholm, Sweden (06-10.09)
2008 Fundamentals of Electronic Nanosystems, St. Petersburg, Russia (28.06-04.07)
2007 SFB research colloquium in Fulda, Germany (29-31.05)
2007 Nanospintronic Design and Realization, Dresden, Germany (21-25.05)
2005 Conference on Strongly Interacting Systems at the Nanoscale, ICTP, Trieste, Italy (08-12.8)

5.5 Scientific visits and stays in institutions

2019 University of Kaiserslautern, Germany (29.04 - 03.05)
2018 Martin Luther University, Halle, Germany (09.07-31.08)
2018 University of Kaiserslautern, Germany (02-05.07)
2018 Martin Luther University, Halle, Germany (30.04-04.05)
2017 University of Kaiserslautern, Germany (01.08-30.09)
2011 Martin Luther University, Halle, Germany (10-14.11)
2007 Rzeszow University of Technology, Poland (13-17.11)

5.6 Participation in schools and conferences without a presentation

2018 30 years of AKLT, Vancouver, Canada
2010 OPTIMAS Workshop, Kaiserslautern, Germany
2005 IOP Theory of Condensed Matter Annual Meeting, Warwick, UK
2004 Theoretical Condensed Matter Physics Summer School, Windsor, UK
2004 Nanophysics: Coherence and Transport, Summer School, Les Houches, France
2003 Theoretical Condensed Matter Physics Summer School, Ambleside, UK
2003 Superconductivity Winter School, Cambridge, UK

5.7 International cooperation

Collaborations:

- Tadeusz Domański, UMCS, Poland
- Maciej Maśka, University of Silesia, Poland
- Michael Fleischhauer, Technical University of Kaiserslautern, Germany
- Levan Chotorlishvili, Martin-Luther University of Halle-Wittenberg, Germany
- Stuart Tessmer, Michigan State University, USA
- Alex Levchenko, University of Wisconsin-Madison, USA
- Cristina Bena, IPhT, CEA Saclay, France
- Pascal Simon, LPS Orsay, France
- Sebastian Eggert, Technical University of Kaiserslautern, Germany
- Jesko Sirker, University of Manitoba, Kanada
- Ian Affleck, University of British Columbia, Kanada
- Florian Gebhard, Philipps-Universität Marburg, Germany
- Vitalii Dugaev, Technical University of Rzeszów, Poland
- Jamal Berakdar, Martin-Luther University of Halle-Wittenberg, Germany
- Igor Yurkevich, Aston University, UK
- Igor Lerner, University of Birmingham, UK

6 Teaching, organisation, and outreach

6.1 Teaching

Conducted classes:

	UMCS, Poland	
2019/2020 W.	Modern Topics in Condensed Matter	PhD Lecture Course
2019/2020 W.	Quantum Liquids	PhD Lecture Course
2019/2020 W.	Nanophotonics	Lecture Course
	Rzeszow University of Technology Poland	
2018/2019 S.	Physics	Laboratory
2018/2019 S.	Higher Mathematics in English II	Lecture course
2018/2019 S.	Calculus and Linear Algebra	Lecture course
2018/2019 S.	Physics I	Lecture course
2018/2019 W.	Differential Equations	Lectures and exams
2018/2019 W.	Physics I	Lectures and exams
2018/2019 W.	Mechanics	Laboratory
2017/2018 S.	Physics II	Lectures and exams
2017/2018 S.	Physics II	Exercises and exams
2017/2018 S.	Higher Mathematics in English II	Lectures and exams
2017/2018 W.	Mechanics	Laboratory
2017/2018 W.	Linear Algebra	Lectures and exams
	Michigan State University, USA	
2017	Calculus I	Lectures and exams
2016	Linear Algebra	Lectures and exams
	University of Kaiserslautern, Germany	
2012	Quantum Many Body Theory	Exercises and exams
2012	Quantum Mechanics	Exercises and exams
2011	Advanced Quantum Mechanics	Exercises and exams
2011	Field Theory in Condensed Matter	Exercises and exams
	Martin-Luther-University, Germany	
2009	Quantum Field Theory	Exercises
	University of Birmingham, UK	
2002-2006	Mathematics for physicists	Exercises
2004	C++	Laboratory

Helping with supervision of PhD students:

- Hamidreza Kazemi 2017-, Department of Physics, University of Kaiserslautern, Germany
- Denis Morath 2011-2016, Department of Physics, University of Kaiserslautern, Germany Title: "Numerical Simulations of Low Dimensional Systems"

Helping with supervision of Masters students theses:

- Philipp Korell, 2011-2012, Department of Physics, University of Kaiserslautern, Germany
- Pia Adam, 2011-2012, Department of Physics, University of Kaiserslautern, Germany

6.2 Conference organisation

On the organising and program committees for the conference: 13th International School on Theoretical Physics: Symmetry and Structural Properties of Condensed Matter, Rzeszów 10-15 September 2018

6.3 Other achievements

- Organiser of PhD Journal Club in the Institute of Physics, UMCS (2019)
- Editor of Proceedings for SSPCM 2018 in Acta Physica Polonica A (2019)
- Administrator of PRz Spintronics website (2018-2019)
- Organiser of spintronics group seminars in the Department of Physics and Medical Engineering in Rzeszów University of technology (2017-2019)
- I am a member of the Institute of Physics (UK)
- I am the author of a blog dedicated to popularising the ideas of condensed matter physics: condensedmattersblog.wordpress.com

7 Other scientific achievements

7.1 Research grants as principal investigator

2019-2020 DAAD/NAWA Bilateral Exchange Grant for "The Effect of Coulomb Interactions on Topological Spin Orbit Torques" jointly between Dr Nicholas Sedlmayr from Rzeszów University of Technology and Prof. Sebastian Eggert from the Technical University of Kaiserslautern

7.2 Refereeing

I have refereed for the following scientific journals:

- Physical Review Letters
- Physical Review B
- Annals of Physics
- Journal of Magnetism and Magnetic Materials
- New Journal of Physics
- Canadian Journal of Physics
- Physica E
- Journal of Physics: Condensed Matter
- Journal of Physics A
- Physical Review Applied

7.3 Other scientific publications

7.3.1 Additional articles, after doctorate, in scientific journals

(P15) **N. Sedlmayr** and J. Berakdar
Transport properties of an interacting quantum dot in a non-uniform magnetization
Europhys. Lett., **83**, 57003 (2008)

(P16) **N. Sedlmayr**, V.K. Dugaev, and J. Berakdar
Current-induced interactions of multiple domain walls in magnetic quantum wires
Phys. Rev. B, **79**, 174422 (2009)

(P17) **N. Sedlmayr**, V.K. Dugaev, and J. Berakdar
Role of non-collinear magnetization: from ferromagnetic nanowires to rings
Physica Status Solidi (b), 247, 2603 (2010)

(P18) **N. Sedlmayr**, V.K. Dugaev, M. Inglot, and J. Berakdar
Indirect interaction of domain walls
Physica Status Solidi RRL, **5**, 450 (2011)

(P19) **N. Sedlmayr**, S. Eggert, and J. Sirker
Electron scattering from domain walls in ferromagnetic Luttinger liquids
Phys. Rev. B, **84**, 024424 (2011)

(P20) **N. Sedlmayr**, V.K. Dugaev, and J. Berakdar
Spin density waves and domain wall interactions in nanowires
Phys. Rev. B, **83**, 174447 (2011)

(P21) **N. Sedlmayr**, J. Ohst, I. Affleck, J. Sirker, and S. Eggert
Transport and scattering in inhomogeneous quantum wires
Phys. Rev. B (Rapid Comm.) **86**, 121302(R) (2012)

(P22) **N. Sedlmayr** and J. Berakdar
Negative differential magnetoresistance in ferromagnetic wires with domain walls
Phys. Rev. B, **86**, 024409 (2012)

(P23) **N. Sedlmayr**, P. Korell, and J. Sirker
Two-band Luttinger liquid with spin-orbit coupling: Applications to monatomic chains on surfaces
Phys. Rev. B., **88**, 195113 (2013)

(P24) **N. Sedlmayr**, P. Adam, and J. Sirker
Theory of the conductance of interacting quantum wires with good contacts and applications to carbon nanotubes
Phys. Rev. B., **87**, 035439 (2013)

(P25) **N. Sedlmayr**, D. Morath, J. Sirker, S. Eggert, and I. Affleck
Conducting fixed points for inhomogeneous quantum wires: a conformally invariant boundary theory
Phys. Rev. B, **89**, 045133 (2014)

(P26) **N. Sedlmayr**, V.K. Dugaev, and J. Berakdar
Dynamics of the polarization of a pinned domain wall in a magnetic nanowire
Physica Status Solidi (b), **251**, 231 (2014)

(P27) D. Morath, **N. Sedlmayr**, J. Sirker, and S. Eggert
Conductance in inhomogeneous quantum wires: Luttinger liquid predictions and quantum Monte Carlo results
Phys. Rev. B, **94**, 115162 (2016)

(P28) R.M. Reeve, A. Loescher, H. Kazemi, B. Dupé, T. Winkler, D. Schönke, J. Miao, K. Litzius, **N. Sedlmayr**, I. Schneider, J. Sinova, S. Eggert, and M. Kläui
Scaling of intrinsic domain wall magneto-resistance with confinement in electromigrated nanocontacts
Phys. Rev. B **99**, 214437 (2019)

(P29) T. Masłowski, and **N. Sedlmayr**
Quasiperiodic dynamical phase transitions in multiband topological insulators and connections with entanglement entropy and fidelity susceptibility
Phys. Rev. B **101**, 014301 (2020)

7.3.2 Conference proceedings after doctorate

(P30) **N. Sedlmayr**, V.K. Dugaev, J. Berakdar, V.R. Vieira, M.A.N. Araújo, and J. Barnaś
Spin and charge transport through non-collinear magnetic nanowires
J. Magn. Magn. Mater., **322**, 1419 (2010)

(P31) **N. Sedlmayr**, S. Eggert, and J. Sirker
Non-collinear ferromagnetic Luttinger liquids
J. Phys.: Conf. Ser., **303**, 012107 (2011)

(P32) M. Sedlmayr, **N. Sedlmayr**, and V.K. Dugaev
Current Induced Dynamics of One-Dimensional Skyrmions
Acta Physica Polonica A, **135**, 1268 (2019)

7.3.3 Lecture notes after doctorate

(P33) **N. Sedlmayr**
Dynamical Phase Transitions in Topological Insulators
Acta Physica Polonica A, **135**, 1191 (2019)

7.3.4 Book chapters (on-line) after doctorate

(P34) **N. Sedlmayr**, J. Berakdar, M.A.N. Araújo, V.K. Dugaev, and J. Barnaś
Charge and spin transport in magnetic nanowires
Nanowires - Fundamental Research (Intech, Croatia) (2011)

7.3.5 Articles, before doctorate, in scientific journals

(P35) **N. Sedlmayr**, I.V. Yurkevich, and I.V. Lerner
Tunnelling density of states at Coulomb blockade peaks
Europhys. Lett., **76**, 109 (2006)

7.4 Results in other areas outside of the habilitation

In addition I have been heavily involved in several other fields of condensed matter physics since my doctorate. The first is transport in nanowires. The second is the dynamics of domain walls in ferromagnetic wires. In particular how domain walls can interact with each other due to the influences of the electronic degrees of freedom on these systems.

A one dimensional wire of interacting fermions has low energy bosonic excitations, describing collective excitations of the fermions. However what happens in inhomogeneous systems, when there is no translational or scale invariance, is not obvious. These regions of inhomogeneity cause intrinsic backscattering of particles, which is a relevant perturbation to the system, leading to an insulating fixed point at low energies. We have shown that in some models this backscattering can be tuned to zero by a simple matching of velocities of the bosonic excitations in the wire and leads (P21). This leads to a conducting fixed point described by a completely new type of conformally invariant boundary theory (P25), which corresponds to neither the usual periodic nor open boundary condition. I have also been able to show by comparing a low-energy field theory with numerical simulations that such inhomogeneous wires can indeed be described by a Luttinger liquid theory to a high accuracy, even for very large jumps in parameters and strong interactions (P21), (P25), and (P27).

Realisations of one dimensional systems include carbon nanotubes and gold atoms deposited on Germanium surfaces. Both of these particular examples have in fact more than one conduction band at the Fermi energy. Such multi band models exhibit different phases compared with the usual Luttinger liquid. I have modelled the gold atom wires and investigated their phase diagram as a function of interaction strength and magnetic anisotropy, finding that despite the many possible scattering processes in such a system a Luttinger liquid has still exists (P23). Indications of this phase have been found experimentally. I have also calculated the temperature and wire length dependent effect on transport due to scattering from phonons in carbon nanotubes when the interactions between the electrons are taken into account, and compared the results to transport experiments and previous calculations (P24). This requires a working theory of the conducting fixed point, generalised to take into account scattering events within the carbon nanotube which we developed.

Spin-charge separation is one of the hallmarks of the Luttinger liquid, but is destroyed by an applied magnetic field, giving rise to excitations which mix the spin and charge degrees of freedom. Furthermore ferromagnetic wires containing domain walls have recently given rise to a large amount of work focusing on the motion of these domain walls caused by the scattering of conduction electrons. I have combined these ideas to analyse the effect of electron-electron interactions on the magnetic and electronic behaviour of such wires in the one-dimensional limit. For the electronic degrees of freedom we have a Luttinger liquid with the domain wall playing the role of an extended magnetic impurity. This impurity can cause scattering of the normal modes (no longer spin and charge) of the Luttinger liquid and we identify a range of possible phases in the low energy limit. Of particular interest is a spin gapped phase which implies that the motion of the domain wall would be suppressed in this phase (P19).

The current induced motion of ferromagnetic domain walls in wires is caused by the spin dependent scattering from the domain wall of the conduction electrons. I have demonstrated that this scattering can also cause a current induced interaction between the domain walls mediated by the electrons (P16) and (P20). This interaction complicates the domain wall motion and in extreme circumstances can destroy the coherent motion of the domain wall. Even in the absence of a current there is an analogue of the RKKY interaction between the domain walls in a wire (P17), (P18), (P22), and (P26).